TESTING THE SOLOW MODEL IN NIGERIA'S ECONOMY

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Abstract
This study empirically tests the Solow model using the Nigeria's economy as a case study. The study utilized the ordinary least square techniques and annual time series data spanning the years (1970-2012). The Basic Solow model and the Augmented Solow model were estimated. While the Basic Solow model was completely validated using the Nigeria's economy, the Augmented Solow model was non-compliant *with prescriptions by Romer, Mankiw & Weil. The study recommended among others the creation of enabling environment for an effective macroeconomic policy framework that supports the Solow model, policy consistency and the integration of Solow variables into policy formulation.*

Keywords: Basic Solow Model, Augmented Solow model, OLS, Nigerian economy

Introduction
Economic growth has occupied a central place in public economic policy over the years. The primary objective of policy formulation and implementation is to attain a path of sustainable economic growth. The keen interest on economic growth is premised on the understanding that a well articulated and managed growth will help redress the high prevalent rate of poverty among nations. Evidence abound of countries that have redressed poverty by stimulating economic growth. For instance, during the 90s, China and Indian experienced robust growth and consistently transited many from low income to middle income group (Inderwildi et al., 2014). Crafts (2000) emphasizes that since World War II, the growth of real GDP has become a key policy objective in almost all countries. Instructively, it is imperative for policy makers to understand the determinants of growth in order to judiciously use policy to influence the time path of economic growth.

Solow (1956) is among the first persons that conducted studies to find out the long-run growth path; employing the neoclassical production function with its well-behaved isoquant and implied assumptions of homogenous good, exogenous labour-augmenting technical progress, full employment and exogenous growth rate of labour force, he presented a model where per capita output is dictated by the exogenous saving rate and population growth rate at least in the short-run. However, when the economy attains steady state, there will be no room for further growth in per capita output in the absence of exogenous shock. The model decomposed the growth in per capita income into the component accounted for by both savings rate and populational growth and the other component accounted for by the Solow's residual otherwise known as "technical change". In what follows, the steady state of countries is determined by their savings rate and populational growth rate. All other things being equal, the higher the savings rate, the higher the growth rate of per capita income , while the higher the populational growth rate, the lower the growth rate of income per capita .

Furthermore, the Solow model has showed that persistent growth must come from technical progress but not savings. Thus, savings can only stimulate growth temporally, but diminishing returns to capital will eventually set in as the economy converges on its steady state, where growth can only be stimulated by technical progress. The Solow Model has enjoyed wide applications. for instance, it was first applied to the USA economy and the exercise yielded fairly robust outcome. However, recent growth theorists refuted the Solow model in favour of the endogenous growth model, which assumes constant and increasing returns to capital. However, the Solow model gained a grand comeback when Romer, Mankiw and Weil (1992) incorporate human capital as another critical determinant of income. The reframed Solow model tagged "Augmented Solow Model" presented the sources of income growth as savings rate, human capital, population growth and technological progress (see Mankiw, Romer & Weil, 1992). Studies such as Ding and Knight (2009) and Mankiw, Romer & Weil (1992) have also found that if human capital is not accounted for in the model, then the quantitative implications of different saving and population growth rates are biased upward.

Over the years, Nigerian policy makers had pursued diverse growth programmes aimed at attaining sustainable growth and development. Some of these programmes are Structural Adjustment Programme (SAP), National Population Programme (NPP), National Economic Empowerment and Development Strategies (NEEDS), etc. These programmes aim at raising the country's living standard through variety of reforms including macroeconomic stability. These programmes not withstanding, the country has experienced development disaster. Savings have consistently been low. Between 1986 and 1989, domestic savings averaged 15.7% of GDP. With the distress in the financial sector, savings fell astronomically so that the savings rate was 6.4% by the year 2004. Furthermore, Nigeria has experienced population explosion over the years. According to the 2012 revision of the world population prospects, the total population was 159,708,000 by the year 2010 compared to only 37,860,000 in 1950. The population growth rate is currently 3.2%, which researchers have lamented it will result in populational explosion that will further worsen unemployment, poverty and environmental pollutions. To this extent, both the poverty rate and unemployment rate have assumed unmanageable proportion with over 70% of the Nigeria's population wallowing in poverty and 23.29% of the population unemployed (NBS, 2012).

Against the backdrop that the Nigeria's growth process is hazy, this paper attempts to examine some of the necessary ingredients for sustained growth in Nigeria using the Solow model as the theoretical framework. This study will borrow the approach utilized by Romer, Mankiw and Weil (1992). This study on broad terms will empirically verify both the basic and the Augmented Solow model using Nigeria's economic growth as the litmus test.

Conceptual issues

There is no doubt that this study raises conceptual issues which require clarification for understanding and smooth presentations. The key concepts in this study are capital accumulation, physical capital, human capital, population growth and technological progress.

Capital accumulation

This is simply the creation of wealth, the gathering of object of value and financial investment made with the intent of making more money in the form of profit, rents, interest, royalties, etc. Two critical issues are central to capital accumulation. They are investment made for the accumulation of new capital and the decumulation of capital through wear and tear, hence, the motion equation for capital is expressed as $K_t = I_t - \alpha K_t$. Basically, there are two types of capital. They are physical and Human capital.

Physical capital

In the parlance of economics, physical capital refers to man-made factors of production deployed in the creation of wealth and it includes machinery, building or computers. Physical capital is one of the four factors of production in economics. Physical capital refers to fixed capital and any kind of real asset that is channeled into the production process.

Human capital

Human capital is simply the accumulation of competence; skills, training, knowledge etc acquire either formally or informally. Human capital is simply labour that is educated and healthy. Human capital has been considered as a critical factor of production. In the Neoclassical growth model, human capital was not considered, however Romer, Mankiw and Weil (1992) integrated human capital into the traditional Solow model and obtained the "Augmented Solow Model". Basically, human capital can be divided into four

(4) categories of Biological, Knowledge, Track Record and social capital. This study deals mostly with the application of the term human capital conceptualized as "knowledge capital" that measures approximately the percentage of the working – age population in secondary school i.e. the secondary school enrolment rate will be used to proxy human capital.

Populational growth

It shows the rate of change of the population over times and can be quantified as the change in the number of individuals of any species in a population using per unit time for measurement. In this study, the term population growth is the average rate of growth of the working-age population. In the parlance of Solow, the labour force grows overtime at the rate n. Thus, populational growth rate is exogenous. Within the content of the neoclassical economics, population growth is negatively related to income per capita (Solow, 1957). This is because population spreads the available capital more thinly to large population, so that it has negative effect on capital stock accumulation.

Theoretical literature

Several theories have attempted to explain the determinants of economic growth. In this section of the study, we shall examine some of the theories ranging from the Neoclassical Growth Models, Endogenous Growth Models and Institutional Growth Models.

Neoclassical growth models

The Neoclassical growth model was elaborated by R.M. (1956) and postulated that output growth is a function of capital accumulation, exogenous labour growth and technological growth. This growth model assumes that labour force grows exogeneously at the constant rate n, technological progress is exogenous and furthermore, the assumptions of diminishing marginal product and constant returns to scale. This model position that it is the populational growth rate and the savings rate that determines the growth rate of per capita income. Given the assumptions of diminishing returns to capital, the economy will converge on its steady state, where the growth rate in capital stock becomes zero i.e. $\dot{K}_t = I_t - \alpha K_t \equiv 0$ and consequently, the growth in per capita income is peaked. Solow (1956 & 1957) argued that at the steady state, no further change in capital stock will move the economy into higher steady state. However, an exogenous shock will be required to pull the economy out. Exogenous shock will take the form of technical progress occasioned by research, education and training. In the Solow model, technical progress is the most important factor as consistent long-run growth in per capita income can only be occasioned by technological progress.

Romer, Mankiw & Weil (1992) introduced the augmented Solow model in which output growth is a function of Physical capital, human capital, exogenous labour growth rate and technological progress. Romer, Weil and Mankiw (1992) experimented by comparing the basic Solow model and the Augmented Solow model. They position that differences in the growth rate of per capita income across countries is accounted for by differences in investment in human capital.

Endogenous growth models

The Basic Solow model came under heavy criticism that the model could not account for cross country differences in per capita income. This shifted the tide in favour of the Endogenous growth model, initiated by Romer (1988). The endogenous growth model unlike the neoclassical growth model disagreed that technological progress is exogenous, but they believe that it is endogenous, and went further to concentrate on the factors that can cause technological progress. Romer (1990) remarked that technological progress is the outcome of knowledge accumulation. This process is considered to be the core element that drives economic growth in the long run. Thus, an economy with knowledge accumulation experiences positive externalities and increasing returns to scale. One of the main postulation of Romer is that in the long-run, the society that has developed science and technology will grow faster than the one that has not. Proponents of the Endogenous growth model recognized the role of human capital investment in the growth process. According to Lucas (1988) and Romer (1990), higher investment in human capital will engender higher growth rate of per capita income.

Institutional approach to growth

This approach to growth sees social infrastructure as the critical determinants of growth. Social infrastructure comprises of the quality of the state legal, political, religious and educational institutions. The presence and absence of these infrastructures dictate the pace of development. It has been argued that a stable rule of law and a healthy investment climate in which property rights are enforced can contribute greatly to economic performance. According to the institutionalists, there are four (4) critical factors that can determine economic performance. They are the institutions, geography, culture and luck. Several studies have been conducted to examine the application of the institutional approach to growth. Glaeser, La Porta, Lopez-de-Silanes and Schleifer (2004) conducted a research to examine whether political institutions can cause economic growth and if human capital and growth leads to institutional improvement. This study utilized government effectiveness and the degree of executive constraints as proxy for institution. The study concluded that institution is not a factor that can stir growth but capital accumulation is. They further concluded that developing nations often experience high growth during dictatorial regime that is effective in promoting beneficial economic policies. Subsequently, as poor countries develop, institutional improvement will take place over time.

An important policy consequence of the Solow growth model is that government programmes design to improve economic performance will only have temporal effects as the economy converge on its steady state. Furthermore, within the Solow's parlance, government policies can only produce level effects on income per capita. However, long-run growth in per capita income can only be predicted by technological change.

A comparative analysis of the different growth models will lend more insight to the discussion so far. While the Solow model offers explanation as to why some countries are doing well while others are not. Firstly, countries with higher savings rate and lower populational growth rate will do better than countries with lower savings rate and higher population growth rate. The former country will experience higher capital per worker, while the latter will experience lower capital per worker. Secondly, the neoclassical growth model explains that technological progress is the only source of long-run growth. However, it does not explain the source of technological progress. The endogenous growth model welded in and relates technical progress to knowledge accumulation. Finally, the institutional approach admonish that government should pursue policies that strengthen economic and educational institutions.

Empirical studies reviewed

In this section of the study, we attempt to look into the history of the field of economic growth in terms of the Solow growth model. In the 1980s, economists began to empirically test the validity of the neoclassical growth model. Baumol (1986) conducted a research to examine one of the propositions of the Solow model – that all countries converge to their steady state. Instructively, because of the presence of diminishing returns to capital, countries with small capital per effective worker will grow faster than countries with higher capital per effective worker. Baumol (1988), Nguyen (1989) and Barro & Salai-rmartin (1991) found strong evidence for this convergence in homogenous regions.

In strong opposition to Baumol (1986), De Long (1988) argued that there was a sample selection bias in Baumal (1986) studies. The opposition is that Baumol (1986) utilized sample of countries that were rich and have successfully developed. In other words, Baumol (1986) samples were homogenous. Accounting for the Bias, De Long (1988) found little evidence for the theory of convergence. This argument was supported by Prichett (1997) and Romer (1987) who also used a diverse sample of countries in their studies, and did not find evidence of convergence.

The inability of some researchers to validate the proposition of convergence within the Solow model shifted the tide in favour of a new growth model known as the Endogenous growth model, which become popular after the writings of Romer (1987) and Lucas (1988). The endogenous growth model unlike the neoclassical growth model postulated that technological progress is endogenous and stimulated by the accumulation of knowledge. Within the new framework, economic growth is not the outcome of capital accumulation but productivity growth, with the assumption of increasing returns to scale (Romer, 1986). It becomes easy for per capita output to increase monotonically over times, unlike the tendency for convergence within the neoclassical growth model.

While endogenous growth model gave a deadly blow to the neoclassical growth model, the neoclassical growth model regained survival under the watch of Romer, Mankiw & Weil (1992) when these three researchers augmented the Solow model by integrating human capital. The augmented Solow model was valid in accounting for differences in cross countries' growth. Further resiliency was gained by the neoclassical growth model when Young (1994) published two articles. Young (1994) studied the growth performance of the newly industrialized countries (NICs) in east Asia found out the robust growth performance of the NICs was accounted for by factor accumulation, but not productivity. This conclusion validated the neoclassical model and helped to push it forward.

Instructively, several empirical researches have been conducted since the seminal work of Romer, Mankiw & Weil (1992). Thus, the Augmented Solow model has gone through rigorous verification. For instance, Islam (1995) criticized their methodological approach in two respects. Firstly, omitted variable bias and secondly, the violation of orthogonality condition. In a recent work, using a large cross-country panel, Hoeffler (2002) use OLS, fixed effect model, first-differenced GMM, system GMM and instrumental variable (IV) approach but found out that GMM estimator better explain the Augmented Solow model. Importantly, when unobserved country effects and endogeneity issues are controlled, Augmented Solow model was validated in explaining sub-Saharan African's growth performance.

Easterly & Levine (2001) and Hall & Jones (1999) adopted the growth accounting approach. The researchers found a large variation in the level of Solow residual and underlie the importance of institutions, government policy and other social infrastructure in determing the cross-country capital accumulation variation. In another study, Ding & Knight (2009) employed a panel data on 146 countries and specifically emphasizes on China to test Augmented Solow model. By adopting the GMM system for the cross-country panel analysis, they found that despite the restrictive assumptions, Augmented Solow model with human capital and structural change account for significant variation in economic growth. In particular, Ding & Knight (2009) examine and discover that rapid economic growth in China is accounted for by the large volume of investment in physical capital, change in employment, structure and output, conditional convergence gain and low population growth policy.

Theoretical framework and methodology

This study is built on the theoretical framework by the basic Solow model advanced by R.M. Solow (1956) and the augmented Solow model advanced by Romer, Mankiw and Weil (1992). In what follows, we present a synoptic review of these models.

The Basic Solow Model

Solow utilizing the neoclassical production function postulated that growth in per capital income is occasioned by physical capital stock (K), the stock of Labour (L) and technical change (A).
Furthermore, labour force grows exogenously at the constant rate, n, hence:

$$\frac{\dot{L}}{L} = n \qquad \text{….. ……………} \qquad (3.1)$$

Also, physical capital stock is simply the amount of national income which is saved and that savings equal investment. Thus, the capital stock equation can be expressed as:

$$\dot{K}(t) = S(t) - \alpha K(t) \qquad \text{……………….} \ (3.2)$$

Where:

$\dot{K}(t)$ = Growth rate of capital stock

S(t) = Savings
Kt = Actual Capital stock
α = Capital depreciation rate

Assuming a Cobb-Douglas Production function:

$$Y(t) = F\left[K(t), L(t)\right] = K(t)^{\alpha}\ L(t)^{1-\alpha}\ O < \alpha < 1 \qquad \text{…. (3.3)}$$

Expressing the production function of (3.3) in terms of per capita income:

$$y(t) = F(k) where k \equiv \frac{K}{L} \qquad \text{…. (3.4)}$$

Recall that a fixed proportion of income is saved, hence:

$$S = sY(t) \qquad \text{… (3.5)}$$

$$\because S = sF(k) \qquad \text{… (3.6)}$$

Re-writing equation (3.2) in per capita terms:

$$\frac{K}{L} = sF(k) - \alpha k \qquad \text{… (3.7)}$$

Given that $K = \frac{K}{L} - nk$ … (3.8)

$\therefore$ Making $\frac{K}{L}$ subject of the formular in equation (3.8)

$$\frac{K}{L} = k + nk \qquad \text{… (3.9)}$$

Equating (3.7) & (3.9)

$$sF(k) - \alpha k = k + nk$$

$$K = sF(k) - \alpha k - nk$$

$$K = sF(k) - (\alpha + n)k \qquad \text{…. (10)}$$

From equation (3.10), three possibilities exist:

- If sF(k) > (α+n)k, K>o ⇨ growth in capital stock is positive
- If sF(k) < (α+n)k, K <o ⇨ growth in capital stock is negative.
- If sF(k) = (α+n)k, K = 0 ⇨ growth in capital stock is zero

In the basic Solow model each country converge on their steady state where $\dot{K} \equiv 0$, determined by their exogenously given savings rate, the population growth rate and the rate of capital depreciation. The assumption of diminishing returns to capital occasioned the steady state. Once the economy converges in its steady state, it will require exogenous shock in the form of technological progress to stimulate any further growth in per capita income. At this junction, we introduce technological shift into the production function:

$$Y(t) = K^{\alpha}(t)\ L^{1-\alpha}(t)\ A(t) \text{ where } o<\alpha<1 \qquad \text{… (3.11)}$$

Where A(t) captures technical shift and it is assumed to grow exogenously at the given rate, g, hence

$$A(t) = A(o)\, e^{gt} \qquad \text{… (3.12)}$$

Thus, the number of effective units of labour A(t) L(t) grows at the rate (n+g). Redefining K as capital per unit of effective Labour k = $\frac{K}{AL}$ and y as income per unit of effective labour so that $y = \frac{Y}{AL}$.

Therefore, the fundamental Solow equation in (3.10) can be re-written:

$$\dot{K}(t) = sF(k) - (n+g+\alpha)K(t) \qquad \text{………. (3.13)}$$

So when k converges on the steady state capital per effective labour denoted as K^* then $\dot{K}(t) \equiv 0$ Therefore, sF(k) = (n+g+α) k^* but F(k) = k^α. Hence, $sK^{*\alpha} = (n+g+\alpha)k^*$ …… (3.14)

From equation (3.14)

$$sk^{*\alpha} = (n+g+\alpha)k^*$$

$$s = (n+g+\alpha)\frac{k^*}{k^{*\alpha}}$$

$$s = (n+g+\alpha)k^{*1-\alpha}$$

$$K^{*1-\alpha} = \frac{s}{n+g+\alpha}$$

$$K^* = \left(\frac{s}{n+g+\alpha}\right)^{\frac{1}{1-\alpha}} \quad \text{….} \quad (3.15)$$

Substituting equation (3.12) and Equation (3.15) into Equation (3.11)

$$Y(t) = K^\alpha(t) \quad A(t) \quad \text{….} \quad (3.11)$$

$$\therefore \quad Y(t) = K^{*\alpha}(t) \quad A(t) \quad \text{….} \quad 3.11)$$

$$y(t) = \left(\frac{s}{n+g+\alpha}\right)^{\frac{\alpha}{1-\alpha}} A(o)\ell^{gt} \quad \text{….} \quad (3.16)$$

Taking the natural logarithm of equation (3.16)

$$L_n Y(t) = \frac{\alpha}{1-\alpha} L_n S - \frac{\alpha}{1-\alpha} L_n (n+g+\alpha) + L_n A(o) + gt \quad \text{….} \quad (3.17)$$

Equation (3.17) presents the central position of the Basic Solow Model, which postulates that growth in per capita income is determined by the savings rate (s), the initial state of technology A(o), the rate of growth of technology (g), the exogenous growth rate of labour force (n) and the rate of capital depreciation (α). Instructively, the Solow model predicts that the share of capital in output (α) is approximately 1/3(Solow, 1956).

The Model further assumes that the elasticity of income per effective workers with respect to the savings rate will be 0.5, but with respect to population growth is -0.5.

Augmented Solow Model

In 1992, Romer, Mankiw and Weil incorporated human capital into the basic Solow model thereby expanding the model. The Augmented Solow model resurrected the neoclassical growth models.

The aggregate production function is given as:

$$Y(t) = K^\alpha(t) L^{1-\alpha-\beta}(t) \quad H^B(t) A(t) \quad \text{….} \quad (3.17)$$

The growth of physical capital per capita is shown below:

K(t) = S_k Y(t) - (n+g+r) K(t) … (3.18)

Where:

S_K = The share of income deployed in the production of physical capital.

The growth of human capital is shown in equation (3.19) below.

h(t) = S_hY(t) - (n+g+r)h(t) …. (3.19)

Where S_h represents the share of income invested into human capital. Both physical and human capital moves towards their steady state values respectively represented as K^* and h^*, and consequently, the growth rate of both factors collapse on zero i.e. $k^*(t) \equiv o$ and $h^*(t) \equiv o$. By Equating (3.18) and (3.19) to zero, we obtain:

$$K^* = \left(\frac{S_k^{1-B} S_h^B}{n+g+r} \right)^{1/1-\alpha-B} \quad \ldots. \quad (3.20)$$

and $$h^* = \left(\frac{S_k^{\alpha}}{n+g+r} S_h^{1-\alpha} \right)^{\frac{1}{1-\alpha-B}} \quad \ldots \quad (3.21)$$

Substituting equations (3.20), (3.21) and (3.12) into equation (3.17)

$$\left(\frac{Yt}{Lt} \right) = \left(\frac{S_k^{1-B} S_h^B}{n+g+r} \right)^{\frac{\alpha}{1-\alpha-B}} \left(\frac{S_k^{\alpha} S_h^{1-\alpha}}{n+g+r} \right)^{\frac{B}{1-\alpha-B}} A(o)\ell^{gt} \quad \ldots. \ (3.22)$$

:. Taking the natural logarithms of equation (3.22)

$$Ln\left[\frac{Yt}{Lt} \right] = Ln\,A(o) + gt + \frac{B}{l-\alpha-B} LnSh + \frac{B}{1-\alpha-B} LnSk = \frac{\alpha}{1-\alpha-B} Ln(n+g+r) \quad \ldots \ (3.23)$$

The augmented Solow Model shows that per capita income growth depends on the initial technical progress A(o), the exogenous growth rate of labour force (n), the technological growth rate (g), human capital (S_h) and physical capital (S_k). Romer, Mankiw and Weil (1992) assumed that the share of physical capital in output is 1/3, the share of human capital lies between 1/3 and 1/2

Model specification

This study will utilize two models. While Model 1 utilizes the Basic Solow Model as theoretical framework, Model II utilizes the human capital Augmented Solow Model. The models are specified below:

Model 1

This model will help us examine the extent to which Nigeria's growth data fit the Basic Solow Model. Utilizing the assumptions that both the rate of technical change (g) and the depreciating rate of capital (α) are exogenous, in short putting (g+r) at 5%, the elasticity of per capita income to saving rate and population growth rate are respectively 0.5 and -0.5, then Equation (3.17) can be expressed in estimatable form, giving rise to Model 1:

$$Ln\left(\frac{Yt}{Lt} \right) = B_o + B_1 LnS + B_2 L_n n + \ell_t \quad \ldots.. \quad (3.24)$$

Where:

$\frac{Yt}{Lt} = Income\ per\ capita$

S = The savings rate (to be proxied by investment rate)

n = the population growth rate

et = Error term

Ln = natural logarithm

$B_{o,}$ B_1 & B_2 = parameter estimates

N.B. Since the model is expressed in logarithms, then B_1 and B_2 are elasticity estimates (see Iyoha, 2006). B_1 and B_2 respectively capture the elasticity of income per capita respectively with respect to savings rate (S) and populational growth rate (n). Based on aprori expectations, B_1 and B_2 will be expected to be 0.5 and -0.5 respectively (solow,1956)

Model II

Model II helps us achieve the second objective of the study. It will be deduced from Equation (3.23). Utilizing the assumptions in the Basic Solow Model: (I). That g and r are exogenous and (g+r) is 5% (II). The elasticity of per capita income with respect to the savings rate is 0.5 (III). The elasticity of per capita income with respect to the populational growth rate is -0.5.

By reformulating Equation (3.23), we obtain Model II:

$$Ln\left(\frac{Yt}{Lt}\right) = \alpha o + \alpha_1 Lnn + \alpha_2\, LnS_h + \alpha_3 LnS_k + et \qquad \ldots. \qquad (3.25)$$

Where:

$\frac{Yt}{Lt}$ = per capital income

S_h = Share of income invested in human capital

S_k = Share of income invested in physical capital

et = Error term

Ln = Natural logarithm

α_0, α_1 & α_3 = parameter estimates

Method of analysis

This study will utilize the ordinary least square techniques in estimating and analyzing the two models. The study will employ the approach utilize by Romer, Mankiw & Weil (1992). Essentially, model I will be evaluated to see whether the sign and sizes of the elasticity estimates conform to their aprori expectations (i.e. $B_1 = 0.5$ & $B_2 = -0.5$). Model II will be evaluated based on same aprori expectations, and in addition, the estimated Model II will be expected to mark improvement over Model I, by possessing a better goodness of fit.

Data and data sources

This study will utilize secondary data which will be obtained from reliable sources such as the Central Bank of Nigeria's Statistical Bulletin, World Development indicators CD-Rom, 2014, pen world table etc. The data are annual time series covering the years (1970-2012). The Data to be utilized are the per capita income, the populational growth rate, investment rate, and secondary school enrolment rate. In this study, populational growth rate is used to proxy the exogenous labour force growth rate, investment rate as a proxy for physical capital and secondary school enrolment rate as a proxy for human capital.

Presentation and analysis of estimated models

Analysis the Basic Solow Model

In the Basic Solow Model, we regressed the GDP per capita on the investment rate and population growth rate. The Model is presented below and subsequently analyzed.

Table 1:The Basic Solow Model

Dependent Variable: GDP PWker

Variables	Coefficients	Std. Error	t-ratios	Prob.
Constant	23.62	1.75	13.52**	0.00
Log (INVRT)	0.56	0.187	2.98**	0.01
Log (POPGR)	-0.64	0.128	-5.03	0.003

R^2 = 0.68

F – Stat = 12.12

DW = 0.699

Implied capital share from α_1 is 0.358.

Implied capital share from $-\alpha_2$ is 0.392.

** Indicates coefficients that are significant at both 1 & 5%

Source: Author's Computation (2015)

The Estimated Basic Model is depicted in table 1. Essentially, three aspects of the results support the Solow Model. First, the estimated model shows that 68% of the variation in GDP Pwker is accounted for by the variations in populational growth rate and the investment rate. Thus, populational growth rate and investment rate are reliable determinants of Nigeria's economic growth performance proxied by the GDPpwker. Secondly, the signs for the coefficients of investment rate and the populatinal growth rate conform to the restrictions by the Basic Solow

Model. Thirdly, the Basic Solow Model has predicted that though the signs of the coefficients for investment rate and populational growth rate should alternate, but the sizes should be the same. The estimated model fulfils this condition with negligible deviation. Finally, the implied share of capital in output is 0.358, which is in the neighbourhood of the prediction that the share should be put at 0.3. Thus, the growth data of Nigeria conform to the Basic Solow Model.

Analyzing the Augmented Solow Model

The Augmented Solow model integrates human capital into the Basic Solow Model. Thus, GDPpwker is regressed on investment rate (INVRT), populational growth rate (POPGR) and human capital proxied by secondary school enrolment (SSER). The Model is presented below and subsequently analyzed.

Table 2: The Augmented Solow Model

Dependent Variable: GDPpwker

Variables	Coefficients	Std. Error	t-ratios	Prob.
Constant	21.43	4.01	5.34**	0.013
Log (INVRT)	1.08	0.19	5.47**	0.012
Log (POPGR)	-0.80	0.16	-5.09**	0.015
Log SSER	0.38	0.43	0.86	0.452

R^2 = 0.90
F – Stat = 9.52
DW = 0.0003

Implied share of capital from α_1 is 0.52.
Implied share of capital from $-\alpha_3$ is 0.74.
Implied share of capital from α_2 is 0.36
Source: Author's Computation (2015)

Following the proposition by Romer, Mankiw & Weil (1992), the introduction of human capital into the Basic Solow Model should improve the Fit of the Basic Solow Model. The Augmented Solow Model depicted in Table 2 shows that human capital variable (SSER) is insignificant yet it is correctly sign. However, the inclusion of human capital has increased the coefficients of both investment rate and populational growth rate. Instructively, by augmenting the Basic Solow Model, the goodness of fit has improved as the R^2 = 0.90, shows that 90% of the variation in GDPpwker is accounted for by the included regressors, while the remaining 10%, which could not be accounted for is due to the included error term. Furthermore, all the coefficients are correctly sign, showing that human capital and physical capital positively impact on income per capita, while population growth rate negatively impact on the economy. However, the coefficients for both populational growth rate and investment rate are significant; hence, these variables remain indispensable in achieving economic growth in Nigeria.

Nevertheless, the outcome of the Augmented Solow model depicted in Table 2 does not conform to the prescriptions by the Augmented Solow model by RMW (1992). For instance, RMW (1992) prescribed that the sum of the implied share of both physical and human capital must be less than 0.7 but greater than 0.6. However, the outcome in table 2 puts the sum of implied share at 0.88. Furthermore, by integrating human capital in the Augmented Solow Model, we succeeded in increasing the share of physical capital from 0.36 to 0.52 and this share of capital lies outside the value of 1/3. Conclusively, while the incorporation of human capital improves the goodness of fit, it however does not conform to the prescriptions in the Augmented Solow Model.

Summary of findings

Utilizing the OLS technique and adopting the approach by Romer, Mankiw & Weil (1992), the findings of this study can be summarized below:

The Basic Solow Model with savings and populational growth fit the Nigeria's data. The Basic Model estimated using Nigeria's data showed that both populational growth and savings proxied by investment rate significantly impact the growth of income per capita; while the coefficient for populational growth was negative that of savings was positive. The restriction placed that both coefficients must alternate in signs, but equal in size was met. Furthermore, the implied share of capital was found to be 0.36, which lies within the restricted value of 1/3.

The Augmented Solow Model which incorporate human capital proxied by secondary school enrolment rate improves the goodness of fit of the model, by the included regressors accounting for 90% of the variation in the dependent variable. This confirms the proposition by Romer, Mankiw & Weil (1992). When human capital was introduced, the coefficients for both populational growth rate and savings rate increase. Though human capital has positive coefficients, but it is statistically insignificant. Furthermore, the outcome of the estimated model travest restrictions place on implied shares of capital. Though, all coefficients are correctly signed but an anomaly was introduced by augmenting the Basic Solow Model.

Recommendations

Based on the estimated and analyzed estimated models, we proffer the following recommendations:

The government should make effort to create an enabling environment for an effective macroeconomic policy framework, for the Solow Model to thrive. Thus, policies that promote peaceful economic climate should be encourage.

The government should integrate the Solow's variables (populational growth rate, savings rate and human capital) into policy formulation.

Effort should be made to encourage policy consistency and eschew frequent policy reversal.

Conclusion

The study has empirically examined the validity in the propositions of Solow Model using Nigeria's data. While the Basic Solow Model was validated in Nigeria's economy, the Augmented Solow Model belies the propositions by Romer, Mankiw & Weil (1992). Though. The Augmented Solow model improve the goodness of fit of the Basic solow model, but key prescriptions were in validated.

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